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$N=2$ superconformal algebra on higher-genus Riemann surfaces

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Abstract. Using the Krichever–Novikov bases and the operator product expansions, we construct the $N=2$ superconformal algebra on a genus-g Riemann surface and the BRST charge corresponding to the superconformal algebra. We also check the nilpotency of the BRST charge, and obtain the critical dimension of spacetime as $D=2$ for the $N=2$ superconformal theory on the higher-genus Riemann surface.

1. Introduction

The superconformal algebras on a higher-genus Riemann surface Σ (or called the Krichever–Novikov superalgebras) and their representations play an important role in the study of superconformal field theory over Σ. The $N=0$ and 1 superconformal algebras [1, 2] on Σ are now relatively well understood, but the $N>1$ superconformal algebras are unknown. In this paper, we begin a study of the $N=2$ superconformal algebra on the genus-g Riemann surface Σ. Then we construct the BRST charge corresponding to the superconformal algebra and check the nilpotency of the BRST charge.

2. KN bases

We consider a compact Riemann surface Σ of genus g with the two distinguished points P_+ and P_- as well as local coordinates z_+ and z_- around them such that $z_\pm(P_\pm)=0$. Krichever and Novikov (KN) [3] have shown that there exist a whole family of meromorphic forms $f_j^{(\lambda,\chi)}$ which are holomorphic everywhere on Σ except possibly for poles or branch points in P_+ and P_-. And $f_j^{(\lambda,\chi)}$ are the bases of the space of the meromorphic forms with the conformal weights λ, called KN bases. The expansions of $f_j^{(\lambda,\chi)}$ near $P_\pm$ can be written as

$$f_j^{(\lambda,\chi)}=z_\pm^{\pm j\pm\chi-S(\lambda)}[1+O(z_\pm)](\mathrm{d}z_\pm)^\lambda \tag{1}$$

where $S(\lambda)=\frac{1}{2}g-\lambda(g-1)$, and χ is a real parameter. The index j in (1) takes integer (half-integer) values when g is even (odd). For different values of (λ,χ), one can obtain the meromorphic vector fields $e_j=f_j^{(-1,0)}$, the meromorphic functions $A_j=f_j^{(0,0)}$, the one-differentials $\omega_j=f_j^{(1,0)}$, and the quadratic-differentials $\Omega_j=f_j^{(2,0)}$. In order to describe the fermionic sector, we also need the meromorphic spinor fields $g_\alpha=f_\alpha^{(-1/2,0)}$,

$\frac{3}{2}$-differentials $k_\alpha = f^{(3/2,0)}_{-\alpha}$, $\frac{1}{2}$-differentials $h_{-r} = f^{(1/2,0)}_{r}$. They satisfy the following duality relations,

$$\frac{1}{2\pi i}\oint_{C_\tau} e_i(Q)\Omega_j(Q) = \delta_{ij} \qquad \frac{1}{2\pi i}\oint_{C_\tau} A_i(Q)\omega_j(Q) = \delta_{ij}$$
$$\frac{1}{2\pi i}\oint_{C_\tau} g_\alpha(Q)k_\beta(Q) = \delta_{\alpha\beta} \qquad \frac{1}{2\pi i}\oint_{C_\tau} h_\alpha(Q)h^+_\beta(Q) = \delta_{\alpha\beta}. \tag{2}$$

where $h^+_\alpha(Q) = h_{-\alpha}(Q)$ and $Q \in \Sigma$. The contours $C_\tau = \{Q \in \Sigma, \tau(Q) = \tau\}$ are level lines of the univalent function $\tau(Q) = \mathrm{Re}\int_{Q_0}^{Q} \mathrm{d}p$, where $\mathrm{d}p$, the third kind of differential on Σ with poles of the first order at the points $P_\pm$ with residues ± 1, and Q_0 an arbitrary initial point, and as $\tau \to \pm\infty$, the contours C_τ become circles enveloping the points $P_\pm$.

3. $N = 2$ superconformal algebra on Σ

The $N = 2$ superconformal algebra on the higher-genus Riemann surface Σ is generated by the energy-momentum tensor $T(z)$ and its super-partner $G^i(z)$ ($i = 1, 2$) and $H(z)$ with the conformal weights 2, 3/2 and 1, respectively. $T(z)$, $G^i(z)$ and $H(z)$ can be expanded on the KN bases over Σ:

$$T(z) = \sum_n L_n \Omega^n(z) \tag{3a}$$

$$G^i(z) = \tfrac{1}{2}\sum_\alpha G^i_\alpha(z) k^\alpha(z) \tag{3b}$$

$$H(z) = \sum_n H_n \omega^n(z). \tag{3c}$$

From (2) and (3), we can obtain the generators of the $N = 2$ superconformal algebra on Σ:

$$L_n = \frac{1}{2\pi i}\oint_{C_\tau} \mathrm{d}z\, e_n(z) T(z) \tag{4a}$$

$$G^i_\alpha = \frac{2}{2\pi i}\oint_{C_\tau} \mathrm{d}z\, g_\alpha(z) G^i(z) \tag{4b}$$

$$H_n = \frac{1}{2\pi i}\oint_{C_\tau} \mathrm{d}z\, A_n(z) H(z) \tag{4c}$$

where we have used the following notation

$$A_n(z) = z^{n-1/2g}[1 + O(z)] \tag{5a}$$

$$\omega_n(z) = z^{-n+1/2g-1}[1 + O(z)] \tag{5b}$$

$$g_\alpha(z) = z^{\alpha-g+1/2}[1 + O(z)] \tag{5c}$$

$$k_\alpha(z) = z^{-\alpha+g-3/2}[1 + O(z)] \tag{5d}$$

$$e_n(z) = z^{n-g_0-2}[1 + O(z)] \tag{5e}$$

$$\Omega_n(z) = z^{n-g_0+1}[1 + O(z)] \tag{5f}$$

$$h^+_r(z) = h_{-r}(z) = z^{r-1/2}[1 + O(z)] \tag{5g}$$

where $g_0 = 3g/2$, and z is the local coordinate in the neighbourhood of the point P_+

that vanishes at P_+. (Note that one can also choose a local coordinate z_- that vanishes at P_-.)

In the conformal field theory over a genus-zero Riemann surface [4, 5], a (anti-)commutator can be expressed equivalently as a complex contour integral. Generalizing it to the higher-genus Riemann surface, we have

$$[L_n, L_m] = \oint_{C_\tau} \oint_{C_w} \frac{\mathrm{d}w\,\mathrm{d}z}{(2\pi \mathrm{i})^2} e_m(w) e_n(z) T(z) T(w) \tag{6}$$

$$[L_n, G^i_\alpha] = \oint_{C_\tau} \oint_{C_w} \frac{\mathrm{d}w\,\mathrm{d}z}{(2\pi \mathrm{i})^2} g_\alpha(w) e_n(z) T(z) G^i(w) \tag{7}$$

$$[L_n, H_m] = \oint_{C_\tau} \oint_{C_w} \frac{\mathrm{d}w\,\mathrm{d}z}{(2\pi \mathrm{i})^2} A_m(w) e_n(z) T(z) H(w) \tag{8}$$

$$\{G^i_\alpha, G^j_\beta\} = \oint_{C_\tau} \oint_{C_w} \frac{\mathrm{d}w\,\mathrm{d}z}{(2\pi \mathrm{i})^2} g_\beta(w) g_\alpha(z) G^i(z) G^j(w) \tag{9}$$

$$[G^i_\alpha, H_m] = \oint_{C_\tau} \oint_{C_w} \frac{\mathrm{d}w\,\mathrm{d}z}{(2\pi \mathrm{i})^2} A_m(w) g_\alpha(z) G^i(z) H(w) \tag{10}$$

$$[H_n, H_m] = \oint_{C_\tau} \oint_{C_w} \frac{\mathrm{d}w\,\mathrm{d}z}{(2\pi \mathrm{i})^2} A_m(w) A_n(z) H(z) H(w) \tag{11}$$

where the contours C_w envelop the point w.

It has been shown [6] that the singular part of the operator product expansions (OPE) on the higher-genus Riemann surface Σ is independent of genus g, and only the non-singular part depends on the genus g. We can therefore obtain the OPEs on the Σ from the OPEs [7] on the genus-zero Riemann surface,

$$\begin{aligned}
T(z)T(w) &= \frac{3D}{2(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial_w T(w)}{z-w} + \ldots \\
T(z)G^i(w) &= \frac{3G^i(w)}{2(z-w)^2} + \frac{\partial_w G^i(w)}{z-w} + \ldots \\
T(z)H(w) &= \frac{H(w)}{(z-w)^2} + \frac{\partial_w H(w)}{z-w} + \ldots \\
G^i(z)G^i(w) &= \frac{D}{2(z-w)^3} + \frac{T(w)}{2(z-w)} + \ldots \\
G^1(z)G^2(w) &= \frac{\mathrm{i}H(w)}{(z-w)^2} + \frac{\mathrm{i}\partial_w H(w)}{2(z-w)} + \ldots \\
H(z)H(w) &= \frac{D}{4(z-w)^2} + \ldots \\
H(z)G^1(w) &= \frac{\mathrm{i}G^2(w)}{2(z-w)} + \ldots \\
H(z)G^2(w) &= -\frac{\mathrm{i}G^1(w)}{2(z-w)} + \ldots
\end{aligned} \tag{12}$$

where the dots stand for the terms which are finite as $z \to w$, and these terms are g-dependent, while they do not contribute to the integrals in equations (6)–(10).

Substituting the above OPEs into (6)–(10), and using the following relations about the KN bases on Σ:

$$
\begin{aligned}
& e_n(w)e'_m(w) - e'_n(w)e_m(w) = \sum_{s=-g_0}^{g_0} C^l_{nm} e_{n+m-l}(w) \\
& \tfrac{1}{2}g_\alpha(w)e'_m(w) - g'_\alpha(w)e_m(w) = \sum_{\beta=-g_0}^{g_0} H^\beta_{\alpha m} g_{\alpha+m-\beta}(w) \\
& -A'_n(w)e_m(w) = \sum_{l=-g_0}^{g_0} B^l_{nm} A_{n+m-l}(w) \\
& 2g_r(w)g_s(w) = \sum_{l=-g/2}^{g/2} E^l_{rs} e_{r+s-l}(w) \\
& 2\mathrm{i}[g_s(w)g'_r(w) - g'_s(w)g_r(w)] = \sum_{l=-g_0}^{g_0} D^l_{rs} A_{r+s-l}(w) \\
& 2\mathrm{i}g_r(w)A_m(w) = \sum_{l=-g/2}^{g/2} F^l_{rm} g_{r+m-l}(w)
\end{aligned}
\tag{13}
$$

we obtain the $N=2$ superconformal algebra on Σ:

$$
\begin{aligned}
& [L_n, L_m] = \sum_{l=-g_0}^{g_0} C^l_{nm} L_{n+m-l} + \frac{D}{4}\chi_{nm} \\
& [L_m, G^i_\alpha] = \sum_{\beta=-g_0}^{g_0} H^\beta_{\alpha m} G^i_{\alpha+m-\beta} \qquad (i=1,2) \\
& [L_m, H_n] = \sum_{l=-g_0}^{g_0} B^l_{nm} H_{n+m-l} \\
& \{G^i_r, G^i_s\} = \sum_{l=-g/2}^{g/2} E^l_{rs} L_{r+s-l} + D\varphi_{rs} \qquad \text{(no sum on } i) \\
& \{G^1_r, G^2_s\} = \sum_{l=-g_0}^{g_0} D^l_{rs} H_{r+s-l} \qquad [H_m, H_n] = \frac{D}{4} a_{nm} \\
& [H_m, G^1_r] = \sum_{l=-g/2}^{g/2} F^l_{rm} G^2_{r+m-l} \qquad [H_m, G^2_r] = -\sum_{l=-g/2}^{g/2} F^l_{rm} G^1_{r+m-l}
\end{aligned}
\tag{14}
$$

where the structure constants are given by

$$
C^l_{nm} = \oint_{C_\tau} \mathrm{d}w [e_n(w)e'_m(w) - e'_n(w)e_m(w)] \Omega_{n+m-l}(w)
$$

$$
H^\beta_{\alpha m} = \oint_{C_\tau} \mathrm{d}w [\tfrac{1}{2}g_\alpha(w)e'_m(w) - g'_\alpha(w)e_m(w)] k_{\alpha+m-\beta}(w)
$$

$$
\begin{aligned}
B^l_{nm} &= -\oint_{C_\tau} \mathrm{d}w\, A'_n(w) e_m(w) \omega_{n+m-l}(w) \\
E^l_{rs} &= 2\oint_{C_\tau} \mathrm{d}w\, g_r(w) g_s(w) \Omega_{r+s-l}(w) \\
D^l_{rs} &= 2\mathrm{i}\oint_{C_\tau} \mathrm{d}w [g_s(w) g'_r(w) - g'_s(w) g_r(w)] \omega_{r+s-l}(w) \\
F^l_{rm} &= 2\mathrm{i}\oint_{C_\tau} \mathrm{d}w\, g_r(w) A_m(w) k_{r+m-l}(w)
\end{aligned}
\tag{15}
$$

and the central terms by

$$
\begin{aligned}
\chi_{nm} &= \oint_{C_\tau} \mathrm{d}w\, e_n(w) e'''_m(w) \qquad \varphi_{rs} = \oint_{C_\tau} \mathrm{d}w\, g''_r(w) g_s(w) \\
a_{nm} &= \oint_{C_\tau} \mathrm{d}w\, A_n(w) A'_m(w).
\end{aligned}
\tag{16}
$$

In particular, setting $g=0$ in (15) and (16) one obtains

$$
\begin{aligned}
C^l_{nm} &= (n-m)\delta_{l,0} \qquad & H^\beta_{\alpha m} &= (\tfrac{1}{2}m - \alpha)\delta_{\beta,0} \\
B^l_{nm} &= -n\delta_{l,0} & E^l_{rs} &= 2\delta_{l,0} \\
D^l_{rs} &= 2\mathrm{i}(r-s)\delta_{l,0} & F^l_{rm} &= \tfrac{1}{2}\mathrm{i}\delta_{l,0}
\end{aligned}
\tag{17}
$$

and

$$
\chi_{nm} = \tfrac{1}{4}(n^3 - n)\delta_{m+n,0} \qquad \varphi_{rs} = D(r^2 - \tfrac{1}{4}) \qquad a_{nm} = m\delta_{n+m,0}. \tag{18}
$$

Substituting the structure constants and central terms into (14) we obtain the following superalgebra,

$$
\begin{aligned}
&[L_n, L_m] = (n-m)L_{n+m} + \tfrac{1}{4}D(n^3 - n)\delta_{n+m,0} \\
&[L_m, G^i_\alpha] = (\tfrac{1}{2}m - \alpha)G^i_{m+\alpha} \qquad (i = 1, 2) \\
&[L_m, H_n] = -nH_{m+n} \\
&\{G^i_r, G^i_s\} = 2L_{r+s} + D(r^2 - \tfrac{1}{4})\delta_{r+s,0} \qquad \text{(no sum on } i) \\
&\{G^1_r, G^2_s\} = 2\mathrm{i}(r-s)H_{r+s} \\
&[H_n, H_m] = \tfrac{1}{4}Dm\delta_{n+m,0} \\
&[H_m, G^1_r] = \tfrac{1}{2}\mathrm{i}G^2_{m+r} \\
&[H_m, G^2_r] = -\tfrac{1}{2}\mathrm{i}G^1_{m+r}.
\end{aligned}
\tag{19}
$$

This is just the well known $N=2$ superconformal algebra on a $g=0$ Riemann surface (i.e. the $N=2$ super-Virasoro algebra) [9, 10].

4. BRST charge

In order to quantize a system with the $N=2$ superconformal algebra on the higher-genus Riemann surface, we first construct a BRST charge. Following the method in [8], we define a BRST operator on Σ corresponding to the $N=2$ superconformal algebra (14),

$$\begin{aligned}Q_B=&\sum_n :L_n\eta_{-n}:+\sum_r :G^i_r\rho^i_{-r}:+\sum_n :H_n\hat{\eta}_{-n}:+\tfrac{1}{2}\sum_{n,m}\sum_{l=-g_0}^{g_0} C^l_{nm}:P_{n+m-l}\eta_{-m}\eta_{-n}:\\&+\sum_{\alpha,m}\sum_{\beta=-g_0}^{g_0} H^{\beta}_{\alpha m}:R^i_{\alpha+m-\beta}\rho^i_{-\alpha}\eta_{-m}:+\sum_{n,m}\sum_{l=-g_0}^{g_0} B^l_{nm}:\hat{P}_{n+m-l}\hat{\eta}_{-n}\eta_{-m}:\\&-\tfrac{1}{2}\sum_{r,s}\sum_{l=-g/2}^{g/2} E^l_{rs}:P_{r+s-1}\rho^i_{-s}\rho^i_{-r}:-\sum_{r,s}\sum_{l=-g_0}^{g_0} D^l_{rs}:\hat{P}_{r+s-l}\rho^2_{-s}\rho^1_{-r}:\\&+\sum_{r,m}\sum_{l=-g/2}^{g/2} F^l_{rm}:R^2_{r+m-l}\rho^1_{-r}\hat{\eta}_{-m}:\\&-\sum_{r,m}\sum_{l=-g/2}^{g/2} F^l_{rm}:R^1_{r+m-1}\rho^2_{-r}\hat{\rho}_{-m}:-\alpha\eta_{-g_0}.\end{aligned}\tag{20}$$

Here the constant α is taking into account the ambiguity in normal ordering of operators. As η_n, P_m, $\hat{\eta}_n$, $\hat{P}_m$ and ρ^i_r, R^i_s are the conformal and superconformal ghosts on the Riemann surface of genus g, respectively, they obey the following anticommutation and commutation relations

$$\{\eta_n, P_m\}=\delta_{n+m,0}\qquad \{\hat{\eta}_n, \hat{P}_m\}=\delta_{n+m,0}\qquad [\rho^i_r, R^j_s]=\delta^{ij}\delta_{r+s,0}$$

and others vanish.

Next we check the nilpotency of the BRST charge (20). As is well known, the nilpotency of BRST charge is a crucial test of the self-consistency of the BRST quantization procedure in superconformal field theories as well as the quantum self-consistency of superconformal algebras [11]. Since it is difficult to evaluate directly the square of the BRST charge (20), for simplicity, we define two pairs of new operators,

$$\begin{aligned}\hat{L}_n=\{Q_B, P_n\}=L_n&+\sum_m\sum_{l=-g_0}^{g_0} C^l_{nm}:\eta_{-m}P_{n+m-l}:+\sum_\alpha\sum_{\beta=-g_0}^{g_0} H^{\beta}_{\alpha n}:R^i_{\alpha+n-\beta}\rho^i_{-\alpha}:\\&+\sum_m\sum_{l=-g_0}^{g_0} B^l_{mn}:\hat{P}_{m+n-l}\hat{\eta}_{-m}:-\alpha\delta_{n,0}\end{aligned}\tag{21a}$$

$$\begin{aligned}\hat{H}_n=\{Q_B, \hat{P}_n\}=H_n&-\sum_m\sum_{l=-g_0}^{g_0} B^l_{mn}:\hat{P}_{m+n-l}\eta_{-n}:+\sum_r\sum_{l=-g/2}^{g/2} F^l_{rn}: R^2_{r+n-l}\rho^1_{-r}:\\&-\sum_r\sum_{l=-g/2}^{g/2} F^l_{rn}:R^1_{r+n-l}\rho^2_{-r}:\end{aligned}\tag{21b}$$

$$\begin{aligned}\hat{G}^1_r=[Q_B, R^1_r]=G^1_r&+\sum_m\sum_{s=-g_0}^{g_0} H^s_{rm}:R^1_{r+m-s}\eta_{-m}:-\sum_s\sum_{l=-g/2}^{g/2} E^l_{rs}:P_{r+s-l}\rho^l_{-s}:\\&-\sum_s\sum_{l=-g_0}^{g_0} D^l_{rs}:P_{r+s-l}\rho^2_{-s}:+\sum_m\sum_{l=-g/2}^{g/2} F^l_{rm}:R^2_{r+m-l}\hat{\eta}_{-m}:\end{aligned}\tag{21c}$$

$$\begin{aligned}\hat{G}^2_r=[Q_B, R^2_r]=G^2_r&+\sum_m\sum_{s=-g_0}^{g_0} H^s_{rm}:R^2_{r+m-s}\eta_{-m}:-\sum_s\sum_{l=-g/2}^{g/2} E^l_{rs}:P_{r+s-l}\rho^2_{-s}:\\&-\sum_s\sum_{l=-g_0}^{g_0} D^l_{rs}:\hat{P}_{r+s-l}\rho^1_{-s}:-\sum_m\sum_{l=-g/2}^{g/2} F^l_{rm}: R^1_{r+m-l}\hat{\eta}_{-m}:\end{aligned}\tag{21d}$$

Performing a lengthy calculation, we arrive at

$$[\hat{L}_m, \hat{L}_n] = \sum_{l=-g_0}^{g_0} C^l_{nm} \hat{L}_{n+m-l} + (\tfrac{1}{4}D - \tfrac{1}{2})\chi_{nm}$$
$$[\hat{L}_m, \hat{G}^i_r] = \sum_{s=-g_0}^{g_0} H^s_{rm} \hat{G}^i_{r+m-s} \qquad [\hat{L}_m, \hat{H}_n] = \sum_{l=-g_0}^{g_0} B^l_{nm} \hat{H}_{n+m-l} \tag{22a}$$

$$\{\hat{G}^i_r, \hat{G}^i_s\} = \sum_{l=-g/2}^{g/2} E^l_{rs} \hat{L}_{r+s-l} + (D-2)\varphi_{rs} \qquad \text{(no sum on } i\text{)}$$
$$\{\hat{G}^1_r, \hat{G}^2_s\} = \sum_{l=-g_0}^{g_0} D^l_{rs} \hat{H}_{r+s-l} \tag{22b}$$

$$[\hat{H}_m, \hat{H}_n] = (\tfrac{1}{4}D - \tfrac{1}{2})a_{nm}$$
$$[\hat{H}_m, \hat{G}^1_r] = \sum_{l=-g/2}^{g/2} F^l_{rm} \hat{G}^2_{r+m-l} \qquad [\hat{H}_m, \hat{G}^2_r] = -\sum_{l=-g/2}^{g/2} F^l_{rm} \hat{G}^1_{r+m-l}. \tag{22c}$$

As is well known, the nilpotent condition for the BRST charge Q_B, $Q_B^2 = 0$, is equivalent to the BRST invariance of $\hat{L}_n$, $\hat{H}_n$ and $\hat{G}^i_r$. This in turn implies that they should satisfy the superalgebra without the central extension terms:

$$[\hat{L}_m, \hat{L}_n] = \sum_{l=-g_0}^{g_0} C^l_{nm} \hat{L}_{n+m-l} \qquad [\hat{L}_m, \hat{G}^i_r] = \sum_{s=-g_0}^{g_0} H^s_{rm} \hat{G}^i_{r+m-s}$$

$$[\hat{L}_m, \hat{H}_n] = \sum_{l=-g_0}^{g_0} B^l_{nm} \hat{H}_{n+m-l} \qquad \{\hat{G}^i_r, \hat{G}^i_s\} = \sum_{l=-g/2}^{g/2} E^l_{rs} \hat{L}_{r+s-l} \qquad \text{(no sum on } i\text{)}$$

$$\{\hat{G}^1_r, \hat{G}^2_s\} = \sum_{l=-g_0}^{g_0} D^l_{rs} \hat{H}_{r+s-l} \qquad [\hat{H}_m, \hat{H}_n] = 0$$

$$[\hat{H}_m, \hat{G}^1_r] = \sum_{s=-g/2}^{g/2} F^s_{rm} \hat{G}^2_{r+m-s} \qquad [\hat{H}_m, \hat{G}^2_r] = -\sum_{s=-g/2}^{g/2} F^s_{rm} \hat{G}^1_{r+m-s}.$$

The above 'anomaly-free' condition poses a strong constraint on the spacetime dimension D. Indeed, by equations (22*a*), (22*b*) and (22*c*) we obtain the critical dimension of spacetime for the $N = 2$ superconformal theory on the genus g Riemann surface as $D = 2$. This shows that the critical spacetime dimension is independent of the genus of the Riemann surface. This is because the conformal anomaly is a short-distance effect. It is also embodied in the singular terms of the operator production expansions in section 3.

5. Concluding remarks

We have constructed the $N = 2$ superconformal algebra on a genus-g Riemann surface and the BRST charge corresponding to the superconformal algebra. We have also checked the nilpotency of the BRST charge, which leads to the critical spacetime dimension $D = 2$ for the $N = 2$ superconformal field theory. When $g = 0$, the well known $N = 2$ superconformal algebra on a trivial Riemann surface is recovered.

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